 **Student's Copy**

**2023**

( NEP—2020 )

( 1st Semester )

# PHYSICS (MAJOR)

## ( Mathematical Physics—I )

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## ( SECTION : A—OBJECTIVE )

( *Marks* : 10 )

Tick (✓) the correct answer in the brackets provided : $1 \times 10 = 10$

**1.** Divergence and curl of a vector field are respectively

(*a*) vector and scalar ( )

(*b*) scalar and scalar ( )

(*c*) vector and vector ( )

(*d*) scalar and vector ( )

**2.** If $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$, which of the following vectors is perpendicular to $\vec{A}$?

(a) $\vec{p} = \hat{i} + 2\hat{j} - \hat{k}$ ( )

(b) $\vec{q} = 2\hat{i} + \hat{j} + \hat{k}$ ( )

(c) $\vec{r} = \hat{i} - \hat{j} + 2\hat{k}$ ( )

(d) $\vec{t} = \hat{i} + \hat{j} + \hat{k}$ ( )

**3.** In spherical polar coordinates, the azimuthal angle ($\phi$) can take the value between

(a) 0 and $\pi$ ( )

(b) 0 and $\pi / 2$ ( )

(c) 0 and $2\pi$ ( )

(d) $\pi / 2$ and $2\pi$ ( )

**4.** The scale factors of spherical polar coordinates are

(a) $1, r, r\sin\theta$ ( )

(b) $1, r, r$ ( )

(c) $1, \rho, r$ ( )

(d) $1, \rho, 1$ ( )

**5.** Which of the following criteria will be most convenient to choose a coordinate system?

*(a)* Distance ( )

*(b)* Geometry ( )

*(c)* Magnitude ( )

*(d)* Intensity ( )

**6.** If $A$ and $B$ are matrices of the same order, then $(AB^T - BA^T)$ is a

*(a)* symmetric matrix ( )

*(b)* skew-symmetric matrix ( )

*(c)* null matrix ( )

*(d)* unit matrix ( )

**7.** If

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

then the value of $A^4$ is

*(a)* $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ( )

*(b)* $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ ( )

*(c)* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ( )

*(d)* $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ( )

**8.** The diagonal elements of a hermitian matrix

(a) are all imaginary ( )

(b) may be real or imaginary ( )

(c) are all zeroes ( )

(d) are all real ( )

**9.** The value of $\Gamma\left(-\frac{1}{2}\right)$ is

(a) $2\sqrt{\pi}$ ( )

(b) $-2\sqrt{\pi}$ ( )

(c) $\sqrt{\pi}$ ( )

(d) $-\sqrt{\pi}$ ( )

**10.** A Fourier series is an expansion of a periodic function of $f(x)$ in terms of

(a) finite sum of sine series ( )

(b) finite sum of cosine series ( )

(c) finite sum of sine and cosine series ( )

(d) infinite sum of sine and cosine series ( )

## ( SECTION : B—SHORT ANSWERS )

( *Marks* : 15 )

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

### UNIT—I

1. Calculate the gradient of the function $\phi(x, y, z) = yz + zx + xy$ at $(2, -1, 2)$.

2. Show that the curl of grad $\phi$ is zero.

### UNIT—II

3. The motion of a particle is represented by $x = 5t^2 - 7$; $y = 7\cos t$ and $z = 3\sin t$. Find the instantaneous velocity from the instantaneous position vector.

4. If $u_i$, where $i = 1, 2, 3, \cdots$ are orthogonal curvilinear coordinates, then prove that $\left|\vec{\nabla} u_i\right| = \frac{1}{h_i}$.

### UNIT—III

5. Show that the matrix $B^{\Theta} AB$ is hermitian or skew-hermitian depending on whether $A$ is hermitian or skew-hermitian.

6. If $A^T$ and $B^T$ are transposes of $A$ and $B$ respectively, then show that $(A + B)^T = A^T + B^T$, where $A$ and $B$ are conformable to addition.

### UNIT—IV

7. For a $\beta$-function, show that $\beta(p, q) = \beta(p+1, q) + \beta(p, q+1)$.

8. Obtain the complex form of Fourier series for the function $f(x) = e^{-x}$ in the interval $-1 < x < 1$.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

### UNIT—I

1. (a) What is the condition for a vector to be solenoidal? Find the value of $n$ for which the vector $r^n \vec{r}$ is solenoidal, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 1+6=7

   (b) If a force $\vec{F} = x^2 y\hat{i} - 2xy^2 \hat{j}$ displaces a particle in the $xy$-plane from (0, 0) to (1, 3) along a curve $y = 4x^2$, then find the work done. 3

2. (a) Find the area of a triangle $ABC$, where $\vec{a} = (5, -1, 1)$, $\vec{b} = (7, -4, 7)$ and $\vec{c} = (1, -6, 10)$ are respectively the position vectors of $A$, $B$, $C$ relative to the origin $O$. 3

   (b) State Gauss' divergence theorem. Using this theorem, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$ and $S$ is the surface of the sphere having centre $(3, -1, 2)$ and radius 3. 1+6=7

### UNIT—II

3. (a) Show that in curvilinear coordinate system, divergence of the vector function $\vec{F}$ is given by

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(F_1 h_2 h_3) + \frac{\partial}{\partial u_2}(F_2 h_3 h_1) + \frac{\partial}{\partial u_3}(F_3 h_1 h_2)\right]$$

6

   (b) Transform the vector $x\hat{i} - 2z\hat{j} + y\hat{k}$ in cylindrical coordinate system. 4

4. (a) Obtain the expressions in plane polar coordinates for the radial and transverse components of velocities and accelerations for a particle moving in a plane. 6

   (b) Calculate the kinetic energy of a particle in terms of spherical polar coordinates. 4

## UNIT—III

5. (a) Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$ 5

(b) Define orthogonal matrix. Show that the product of two orthogonal matrices is also orthogonal. 1+4=5

6. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$ 5

(b) Show that the characteristic roots of a hermitian matrix are all real. 5

## UNIT—IV

7. (a) Prove that

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{2}$$ 5

(b) Show that

$$\Gamma(1+m)\Gamma(1-m) = \frac{m\pi}{\sin m\pi}$$ 5

8. (a) Obtain Fourier series for the function $x^2$ in $-\pi < x < \pi$ and deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$ 3+2=5

(b) Discuss the application of Fourier series in half-wave rectifier, where the current is represented by the function

$$I = \begin{cases} I_0 \sin \omega t; & 0 \le t \le \frac{T}{2} \\ 0; & \frac{T}{2} \le t \le T \end{cases}$$ 5

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