### MATH100 (MAJOR)

## Student's Copy

#### 2023

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(NEP-2020)

#### (1st Semester)

### MATHEMATICS (MAJOR)

#### (Vector Analysis)

Full Marks: 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A-OBJECTIVE)

(Marks: 10)

Tick ( $\checkmark$ ) the correct answer in the brackets provided :

- 1. The unit tangent vector to the space curve  $x = t^2 + 2$ , y = 4t 5,  $z = 2t^2 6t$ at t = 2 is
  - (a)  $\frac{1}{\sqrt{3}}(-2\hat{i}+2\hat{j}+\hat{k})$  ( )
  - (b)  $-\frac{1}{\sqrt{3}}(-2\hat{i}+2\hat{j}+\hat{k})$  ( )
  - (c)  $-\frac{1}{\sqrt{3}}(-2\hat{i}-2\hat{j}+\hat{k})$  ( )
  - (d)  $\frac{1}{\sqrt{3}}(2\hat{i}+2\hat{j}+\hat{k})$  ()

/248

Contd.

 $1 \times 10 = 10$ 

2. Which of the following is correct?

(a) 
$$\vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$$
 ( )  
(b)  $\vec{u} \cdot \frac{d\vec{u}}{dt} = 2 |\vec{u}| \frac{d|\vec{u}|}{dt}$  ( )  
(c)  $2\vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$  ( )

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10,736 4  $-\vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$ đ (g)

For any vector  $\vec{a}$ , div (curl  $\vec{a}$ ) = ø

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If  $\vec{a}$  is any vector and  $\vec{r} = x\hat{a} + y\hat{j} + z\hat{k}$ , then  $(\vec{a} \cdot \nabla)\vec{r}$  is equal to to E

(c) 
$$\vec{a} \times \vec{r}$$
 ( ) (3)  $\vec{a}$  ( )

(d) a . r ı.

If the vector  $\vec{V} = y^2 z \hat{i} + \alpha x y z \hat{j} + x y^2 \hat{k}$  be a conservative vector, then a is 0 (a)

The line integral of the vector function  $\vec{u}(x, y) = 2y\hat{i} + x\hat{j}$  along the straight ø

0 a)

(q) 10 (d) 12 1

(c)

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- 7. For a given vector  $\vec{V}$ , the projection R of a surface S on the zx-plane is
  - (a)  $\iint_{S} \vec{A} \cdot \hat{n} \, dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dx \, dy}{|\hat{n} \cdot \hat{k}|} \qquad ( )$ (b)  $\iint_{S} \vec{A} \cdot \hat{n} \, dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dy \, dz}{|\hat{n} \cdot \hat{i}|} \qquad ( )$ (c)  $\iint_{S} \vec{A} \cdot \hat{n} \, dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dz \, dx}{|\hat{n} \cdot \hat{j}|} \qquad ( )$ 
    - $(d) \quad \iint_{S} \vec{A} \cdot \hat{n} \, dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dx \, dz}{|\hat{n} \cdot \hat{i}|} \qquad (\qquad)$
  - **8.** If V is the volume enclosed by the surface S, then the value of  $\iint_{S} \vec{r} \cdot \hat{n} dS$  is
    - (a) V ( )
    - (b) 2V ( )
    - (c) 3V ( )
    - (d) 4V ()

9. Stokes' theorem is the relation between

- (a) line and surface integrals ( )
- (b) surface and volume integrals ( )
- (c) line and volume integrals ()
- (d) line, surface and volume integrals ()

10. Which one of the following theorems uses curl operation?

- (a) Green's theorem ( )
- (b) Gauss' divergence theorem ( )
- (c) Ampere's law ( )
- (d) Stokes' theorem ( )

# (SECTION : B-SHORT ANSWERS )

( Marks: 15 )

Answer five questions, taking at least one from each Unit :

UNIT-I

3×5=1.

1. Prove that

$$\frac{d}{dt}[\vec{p}\,\vec{q}\,\vec{r}] = \left[\frac{d\vec{p}}{dt}\vec{q}\,\vec{r}\right] + \left[\vec{p}\,\frac{d\vec{q}}{dt}\,\vec{r}\right] + \left[\vec{p}\,\frac{d\vec{q}}{dt}\,\vec{r}\right] + \left[\vec{p}\,\vec{q}\,\frac{d\vec{r}}{dt}\right]$$

where  $\vec{p} \vec{q} \vec{r}$  are functions of t Hence find

$$\frac{d}{dt}\left[\vec{r} \cdot \frac{d}{dt}\right] \vec{r} \cdot \frac{d}{dt} \cdot \frac{d^2 \vec{r}}{dt} \frac{d^2 \vec{r}}{dt^2}$$

A particle moves along the curves  $x = 2t^2$ ,  $y = (t^2 - 4t)$ , z = (3t - 5), where t is the time. Find the velocity and acceleration at t=1. Also, find the direction of motion at the given value of t Ri

## UNIT-II

- Find the value of curl (grad  $\phi$ ), where  $\phi$  is a scalar function. e.
- Find the directional derivative of  $\phi = x^2yz + 4xz^2$  at (1, -2, -1) in the direction  $2\hat{i} - \hat{j} - 2\hat{k}$ . Also, find the magnitude of greatest directional derivative at the same point. 4

- **5.** If  $\vec{F} = 3xg\hat{i} y^2\hat{j}$ , evaluate  $\int \vec{F} \cdot d\vec{r}$ , where C is the curve in the xy-plane,  $y = 2x^2$  from (0, 0) to (1, 2).
- 6. Find the work done when a force  $\vec{F} = (x^2 y^2 + x)\hat{i} (2xy + y)\hat{j}$  moves a particle from (0, 0) to (1, 1) along a parabola  $y^2 = x$ .

/248

7. Evaluate  $\oint (yz \, dx + zx \, dy + xy \, dz)$  by Stokes' theorem, where C is the curve  $x^2 + y^2 = 1, \ z = y^2.$ 

8. Prove that

 $\iint_{V} \frac{dV}{r^2} = \iint_{S} \frac{\vec{r} \cdot \vec{n}}{r^2} dS$ 

# (SECTION : C-DESCRIPTIVE)

( Marks : 50 )

Answer five questions, taking at least one from each Unit :

 $10 \times 5 = 50$ 

UNIT-I

4 **1.** (a) If  $\vec{r}' = \frac{d\vec{r}}{dt}$ ,  $\vec{r}'' = \frac{d^2\vec{r}}{dt^2}$ , then find the value of  $\frac{d^2}{dt^2} [\vec{r} \times (\vec{r}' \times \vec{r}'')]$ .

(b) Prove that

 $\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{a}\times\frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt}\times\vec{b}$ 

from the definition of derivative of a function.

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Find the arc length of the indicated portion of the curve 3

$$(\mathfrak{h}) = (\cos^3 \mathfrak{h})\hat{j} + (\sin^3 \mathfrak{h})\hat{k}, \ 0 \le t \le \frac{\pi}{2}$$
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ŝ **2.** (a) Prove that a necessary and sufficient condition that a proper vector  $\vec{u}$  remains parallel to a fixed line is that  $\vec{u} \times \frac{d\vec{u}}{dt} = \vec{0}$ .

(b) Find the curvature and unit normal vector for the helix

 $\vec{r}(\mathfrak{t}) = (\alpha \cos \mathfrak{t})\hat{\mathfrak{t}} + (\alpha \sin \mathfrak{t})\hat{\mathfrak{t}} + bt\hat{k}, \ \alpha, b \ge 0, \ \alpha^2 + b^2 \neq 0$ 

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- 3. (a) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $r = |\vec{r}|$  and  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  is a constant vert then find div $\left(\frac{\vec{a} \times \vec{r}}{r^n}\right)$ .
- Find the value of n for which the vector  $r^n \vec{r}$  is solenoidal  $w_{\mathrm{he}_{\mathrm{th}}}$  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$ (q)
- Show that div (grad  $r^n$ ) =  $n(n+1)r^{n-2}$ , where  $r = \sqrt{x^2 + y^2 + z^2}$ . Hence 5 show that  $\nabla^2 \left(\frac{1}{r}\right) = 0$ . **4.** (a)
  - (b) Prove that  $\operatorname{curl}(\vec{a} \cdot \vec{b}) = \vec{a} \operatorname{curl} \vec{b} + \vec{b} \operatorname{curl} \vec{a} + (\vec{a} \cdot \nabla)\vec{b} + (\vec{b} \cdot \nabla)\vec{a}$ .

## UNIT-III

cylinder  $y^2 = 8x$ Then z = 6.and Let  $\vec{F} = 2y\hat{i} + \hat{z}\hat{j} + x^2\hat{k}$  and S is the surface of the octant bounded by the planes y = 4evaluate  $\iint \vec{F} \cdot \vec{n} dS$ . in the first **5.** (a)

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- (b) If  $\vec{F} = (2x^2 3z)\hat{i} 2xy\hat{j} 4x\hat{k}$ , evaluate  $\iiint_{i=1}^{n} \nabla \cdot \vec{F}dV$ , where V is the closed region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4.
- **6.** (a) If  $\vec{F} = 2z\hat{i} x\hat{j} + y\hat{k}$ , evaluate  $\iiint \vec{F}dV$ , where V is the region bounded by

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- S the surfaces x = 0, y = 0, x = 2, y = 4,  $z = x^2$ , z = 2.
- S (b) Evaluate  $\iint \vec{A} \cdot \vec{n} \, dS$ , where  $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$  and S is the surface of the sphere  $x^2 + y^2 + z^2 = 1$  which lies in the first octant.

- Verify Stokes' theorem for  $\vec{A} = (2x y)\hat{i} yz^2\hat{j} y^2z\hat{k}$ , where S is the upper surface of the sphere  $x^2 + y^2 + z^2 = 1$  bounded by its projection on xy-plane. **7**. (α)
- evaluate area  $\int [(2x^2 - y^2) dx + (x^2 + y^2) dy], \text{ where } C \text{ is the boundary of the}$ ij apply and hence theorem State Green's (q)

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- ເດ enclosed by the x-axis and the upper half of the circle  $x^2 + y^2 = a^2$ .
  - show that  $\iint_{x} \nabla(x^2 + y^2 + z^2) d\vec{S} = 6V$ , where V is the volume enclosed State Gauss divergence theorem. Use Gauss divergence theorem to by the surface S. 8. (a)
- Verify Stokes' theorem for  $\vec{F} = (x^2 y^2)\hat{i} + 2x\hat{j}$ , around the rectangle bounded by the straight lines x = 0, x = a, y = 0, y = b. (q)

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 (9) <sup>1</sup>/<sub>4</sub> (2) (2)

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2: Which of the following is correct?

$$(a) \quad \vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$$

$$(b) \quad \vec{u} \cdot \frac{d\vec{u}}{dt} = 2 |\vec{u}| \frac{d|\vec{u}|}{dt}$$

$$(c) \quad 2\vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$$

$$(c) \quad 2\vec{u} \cdot \frac{d\vec{u}}{dt} = |\vec{u}| \frac{d|\vec{u}|}{dt}$$

$$(f) \quad (f) \quad$$

**3.** For any vector  $\vec{a}$ , div (curl  $\vec{a}$ ) =

**4.** If  $\vec{a}$  is any vector and  $\vec{r} = \hat{x} + \hat{y} + \hat{x}\hat{k}$ , then  $(\vec{a} \cdot \nabla)\vec{r}$  is equal to

If the vector  $\vec{V} = y^2 z \hat{i} + \alpha x y z \hat{j} + x y^2 \hat{k}$  be a conservative vector, then a is equal to ໌.

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The line integral of the vector function  $\vec{u}(x, y) = 2y\hat{i} + x\hat{j}$  along the straight Ş 2 line from (0, 0) to (2, 4) is œ

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 $\pmb{\tau}$ . For a given vector  $\vec{V}$ , the projection R of a surface S on the zx-plane is

(a) 
$$\iint_{S} \vec{A} \cdot \hat{n} dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dx dy}{|\hat{n} \cdot \hat{k}|}$$
(b) 
$$\iint_{S} \vec{A} \cdot \hat{n} dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dy dz}{|\hat{n} \cdot \hat{i}|}$$
(c) 
$$\iint_{S} \vec{A} \cdot \hat{n} dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dz dx}{|\hat{n} \cdot \hat{j}|}$$
(d) 
$$\iint_{S} \vec{A} \cdot \hat{n} dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dz dz}{|\hat{n} \cdot \hat{j}|}$$
(d) 
$$\iint_{S} \vec{A} \cdot \hat{n} dS = \iint_{R} \vec{A} \cdot \hat{n} \frac{dx dz}{|\hat{n} \cdot \hat{j}|}$$

7 . ndS is **8.** If V is the volume enclosed by the surface S, then the value of  $\int_{-\infty}^{\infty}$ 

(a) V ( ) (b) 2V ( ) (c) 3V ( ) (d) 4V ( ) 9. Stokes' theorem is the relation between

(a) line and surface integrals (

(b) surface and volume integrals (

(c) line and volume integrals (

line, surface and volume integrals (q) Which one of the following theorems uses curl operation? 9

(a) Green's theorem (

(b) Gauss' divergence theorem (

(c) Ampere's law (

(d) Stokes' theorem (

/248

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(SECTION : B-SHORT ANSWERS )

( Marks: 15 )

3×5=15

Answer five questions, taking at least one from each Unit :

UNIT-I

1. Prove that

$$\frac{d}{dt}[\vec{p}\,\vec{q}\,\vec{\tau}] = \left[\frac{d\vec{p}}{dt}\vec{q}\,\vec{\tau}\right] + \left[\vec{p}\,\frac{d\vec{q}}{dt}\vec{\tau}\right] + \left[\vec{p}\,\frac{d\vec{q}}{dt}\vec{\tau}\right] + \left[\vec{p}\,\vec{q}\,\frac{d\vec{\tau}}{dt}\right]$$

where  $\vec{p} \ \vec{q} \ \vec{r}$  are functions of t. Hence find

$$\frac{d}{dt}\left[\vec{r} + \frac{d\vec{r}}{dt} + \frac{d^2\vec{r}}{dt}\right]$$

**2.** A particle moves along the curves  $x = 2t^2$ ,  $y = (t^2 - 4t)$ , z = (3t - 5), where t the is the time. Find the velocity and acceleration at t = 1. Also, find direction of motion at the given value of t.

## UNIT-II

- 3. Find the value of curl (grad \$), where \$ is a scalar function.
- **4.** Find the directional derivative of  $\phi = x^2 yz + 4xz^2$  at (1, -2, -1) in the direction  $2\hat{i} - \hat{j} - 2\hat{k}$ . Also, find the magnitude of greatest directional derivative at the same point.

5. If  $\vec{F} = 3xg\hat{i} - y^2\hat{j}$ , evaluate  $\int \vec{F} \cdot d\vec{r}$ , where C is the curve in the xy-plane,

 $y = 2x^2$  from (0, 0) to (1, 2).

6. Find the work done when a force  $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$  moves a particle from (0, 0) to (1, 1) along a parabola  $y^2 = x$ . [ Contd.

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. Evaluate  $\oint (yz \, dx + zx \, dy + xy \, dz)$  by Stokes' theorem, where C is the curve

 $x^2 + y^2 = 1, \ z = y^2.$ 

3. Prove that

$$\iint_{V} \frac{dV}{r^2} = \iint_{S} \frac{\vec{r} \cdot \vec{n}}{r^2} dS$$

# (SECTION : C-DESCRIPTIVE )

( Marks : 50 )

nswer *five* questions, taking at least *one* from each Unit :

 $10 \times 5 = 50$ 

UNIT-I

1. (a) If  $\vec{r}' = \frac{d\vec{r}'}{dt}$ ,  $\vec{r}'' = \frac{d^2\vec{r}}{dt^2}$ , then find the value of  $\frac{d^2}{dt^2} [\vec{r} \times (\vec{r}' \times \vec{r}'')]$ .

(b) Prove that

$$\frac{1}{tt}(\vec{a} \times \vec{b}) = \vec{a} \times \frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt} \times \vec{b}$$

from the definition of derivative of a function.

Find the arc length of the indicated portion of the curve 3

$$\vec{r}(t) = (\cos^3 t)\hat{j} + (\sin^3 t)\hat{k}, \ 0 \le t \le \frac{\pi}{2}$$

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S 2. (a) Prove that a necessary and sufficient condition that a proper vector  $\vec{u}$  remains parallel to a fixed line is that  $\vec{u} \times \frac{d\vec{u}}{dt} = \vec{0}$ .

(b) Find the curvature and unit normal vector for the helix

 $\vec{r}(\mathfrak{h} = (\alpha \cos \mathfrak{h})^{\tilde{i}} + (\alpha \sin \mathfrak{h})^{\tilde{j}} + bt\hat{k}, \ \alpha, b \ge 0, \ \alpha^2 + b^2 \ne 0$ 

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UNIT-II

- 3. (a) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $r = |\vec{r}|$  and  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  is a constant vector, then find div $\left(\frac{\vec{a} \times \vec{r}}{r^n}\right)$ .
  - Find the value of n for which the vector  $r^n \vec{r}$  is solenoidal  $w_{h_{\rm the}}$  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$ (q)
- 5+1=6 Show that div (grad  $r^n$ ) =  $n(n+1)r^{n-2}$ , where  $r = \sqrt{x^2 + y^2 + z^2}$ . Hence show that  $\nabla^2\left(\frac{1}{r}\right) = 0$ . **4**. (a)
  - (b) Prove that  $\operatorname{curl}(\vec{a} \cdot \vec{b}) = \vec{a} \operatorname{curl} \vec{b} + \vec{b} \operatorname{curl} \vec{a} + (\vec{a} \cdot \nabla)\vec{b} + (\vec{b} \cdot \nabla)\vec{a}$ .

# UNIT-III

- Let  $\vec{F} = 2y\hat{i} \hat{z}\hat{j} + x^2\hat{k}$  and S is the surface of the cylinder  $y^2 = 8x$ and z = 6. Then in the first octant bounded by the planes y = 4evaluate  $\iint_{S} \vec{F} \cdot \vec{n} \, dS$ . 5. (a)
- (b) If  $\vec{F} = (2x^2 3z)\hat{i} 2xy\hat{j} 4x\hat{k}$ , evaluate  $\iiint \nabla \vec{F}dV$ , where V is the
  - closed region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4.

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- 6. (a) If  $\vec{F} = 2z\hat{i} x\hat{j} + y\hat{k}$ , evaluate  $\iiint \vec{F}dV$ , where V is the region bounded by
- 5 the surfaces x = 0, y = 0, x = 2, y = 4,  $z = x^2$ , z = 2. (q)
  - Evaluate  $\iint \vec{A} \cdot \vec{n} \, dS$ , where  $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$  and S is the surface of the sphere  $x^2 + y^2 + z^2 = 1$  which lies in the first octant.

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248

(a) Verify Stokes' theorem for  $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ , where S is the upper surface of the sphere  $x^2 + y^2 + z^2 = 1$  bounded by its projection on xy-plane.

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- evaluate  $\int [(2x^2 - y^2) dx + (x^2 + y^2) dy]$ , where C is the boundary of the area \$ apply it and hence theorem Green's State (q)
- S enclosed by the x-axis and the upper half of the circle  $x^2 + y^2 = a^2$ .
- State Gauss divergence theorem. Use Gauss divergence theorem to show that  $\iint \nabla(x^2 + y^2 + z^2) d\vec{S} = 6V$ , where V is the volume enclosed by the surface S. (a)

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Verify Stokes' theorem for  $\vec{F} = (x^2 - y^2)\hat{i} + 2x\hat{j}$ , around the rectangle bounded by the straight lines x = 0,  $x = \alpha$ , y = 0, y = b. (q)

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