PHY/V/CC/09 **Student's Copy**

2022

(CBCS)

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The function $f(z)\ \ |z|^2$ with $z\ \ x\ \ iy$ is

(a) differentiable for all values of x and y ( )

(b) differentiable for positive values of x and y ( )

(c) differentiable only at x 0 and y 0 ( )

(d) differentiable only at x 1 and y 1 ( )

2. The value of the integral $_C\, e^{1/z} dz$, where C is the circle $|z|$ 1 is

(a) 0 ( ) *(b)* 2 i ( )

(c) i ( ) *(d)* 2 ( )

3. The solution of the differential equation $y \quad n^2 y \quad 0$; where $y \quad \frac{dy}{dx}$ is

(a) $A e^{nx} \quad B e^{\ nx}$ ( )

(b) $A e^{\ nx} \quad B e^{nx}$ ( )

(c) $A e^{nx} \quad B e^{nx}$ ( )

(d) $A e^{\ nx} \quad B e^{\ nx}$ ( )

4. The displacement function for a string of length l fixed at both ends with zero initial velocity is given by

$$u(x, t) \quad {}_{n \ 1} \ C_n \cos \frac{n \ ct}{l} \quad D_n \sin \frac{n \ ct}{l} \quad \sin \frac{n}{l} x$$

then

(a) $C_n \quad 0$ ( )

(b) $C_n \quad D_n \quad 0$ ( )

(c) $C_n \quad D_n \quad 0$ ( )

(d) $D_n \quad 0$ ( )

5. For integral values of n, $J_n(\ x)$ is

(a) $(\ 1)^n J_n(x)$ ( )

(b) $J_n(x)$ ( )

(c) 0 ( )

(d) $(\ 1)^n \quad J_n(x)$ ( )

6. The value of ${}_1^1 (2x \quad 1) P_3(x)\, dx$ is

(a) 1 ( )

(b) $\ 1$ ( )

(c) 2 ( )

(d) 0 ( )

7. The Fourier transform of $\frac{df}{dt}$, i.e., FT $\frac{df}{dt}$ is

(a) $\frac{}{\sqrt{2}} \quad f(t) e^{\ i \ t} dt$ ( )

(b) $\sqrt{\frac{}{2}} \quad f(t) e^{i \ t} dt$ ( )

(c) $\frac{i}{\sqrt{2}} \quad f(t) e^{\ i \ t} dt$ ( )

(d) $\frac{1}{i \ \sqrt{2}} \quad f(t) e^{i \ t} dt$ ( )

8. If $g(\ )$ be the Fourier transform of $f(t)$, then FT $[t\,f(t)]$ is

(a) $\dfrac{dg}{d}$ ( )

(b) $\dfrac{dg}{d}$ ( )

(c) $i\dfrac{dg}{d}$ ( )

(d) $i\dfrac{dg}{d}$ ( )

9. Laplace transform of $t^{\frac{1}{2}}$ is

(a) $\dfrac{\sqrt{\ }}{s}$ ( )

(b) $\sqrt{\dfrac{\ }{s}}$ ( )

(c) $\dfrac{s}{\sqrt{\ }}$ ( )

(d) $\dfrac{1}{\sqrt{\ s}}$ ( )

10. The inverse Laplace transform of $\dfrac{1}{s^2(s^2 \quad ^2)}$ is

(a) $\dfrac{1}{\ ^3}\{\ t \quad \sin \quad t\}$ ( )

(b) $\dfrac{1}{\ ^3}\{\sin \quad t \quad t\}$ ( )

(c) $\dfrac{1}{\ ^3}\{\sinh \quad t \quad t\}$ ( )

(d) $\dfrac{1}{\ ^3}\{\ t \quad \sinh \quad t\}$ ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Evaluate $\; _C \frac{1}{\sin 2z} dz$, where C is the circle $|z| \quad 1$.

OR

2. Compute the residue of $\frac{\sin z}{1 \quad z^4}$ at $z \quad i$.

UNIT—II

3. Obtain an expression for the displacement function $u(x, t)$ for a string of length l fixed at both ends and released with zero initial velocity.

OR

4. Find the regular singular points of the differential equation $(1 \quad x^2)y \quad 2xy \quad l(l \quad 1)y \quad 0$.

UNIT—III

5. Show that $(n \quad 1)P_{n \quad 1}(x) \quad xP_{n \quad 1}(x) \quad P_n(x)$.

OR

6. Using the generating function for Hermite polynomials, show that

$$H_n(x) \quad (\quad 1)^n e^{x^2} \frac{d^n}{dx^n}(e^{\quad x^2})$$

UNIT—IV

7. For the square wave function given by

$$y \quad a; \; t \quad 0 \text{ to } t \quad \frac{T}{2}$$
$$a; \; t \quad \frac{T}{2} \text{ to } t \quad T$$

Show that the Fourier coefficient $b_n \quad 0$ for all values of n.

OR

8. Show that the Fourier transform of delta function (t) is $\frac{1}{\sqrt{2}}$.

UNIT—V

9. Find Laplace transform of a square wave function

$$F(t) \quad \begin{matrix} a \text{ for } 0 \quad t \quad \frac{T}{2} \\ 0 \text{ for } \frac{T}{2} \quad t \quad T \end{matrix} \; ; \; F(t \quad T) \quad F(t)$$

OR

10. If $f(s) \quad \mathscr{L}[F(t)]$, then show that $\mathscr{L} \; \frac{F(t)}{t} \qquad {}_s \, f(s)\, ds$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* State and prove the necessary condition for analyticity of a complex function. 1+3=4

(b) Find the Laurent's series expansion for $f(z) \quad \frac{z^2 \quad 1}{(z \quad 2)(z \quad 3)}$ valid in the region $2 \quad |z| \quad 3$. 3

(c) Use residue theorem to evaluate the integral ${}_C \frac{4 \quad 3z}{z(z \quad 1)(z \quad 2)} dz$, where C is the circle $|z| \quad \frac{3}{2}$. 3

OR

2. *(a)* State and prove Cauchy's residue theorem. 1+6=7

(b) Use Cauchy's integral formula to evaluate ${}_C \frac{2 \quad z}{z(2 \quad z)} dz$, where C is the circle $|z| \quad 1$. 3

UNIT—II

3. *(a)* Find the power series solution of $(1-x^2)y'' - 2xy' + 2y = 0$ about $x = 0$. 4

(b) Use the method of separation of variables to solve the partial differential equation of heat flow in one dimension $\frac{\partial u}{\partial t} = h^2 \frac{\partial^2 u}{\partial x^2}$ with initial condition $u(x, 0) = \cos 2x$. 4

(c) Find the solution of indicial equation for the differential equation

$$y'' + \frac{1}{2x}y' - \frac{(1+x^2)}{2x^2}y = 0$$ 2

OR

4. *(a)* Find the power series solutions of the differential equation $\frac{d^2y}{dx^2} - \frac{2x}{1-x^2}\frac{dy}{dx} + \frac{2}{1-x^2}y = 0$, about $x = 1$. 6

(b) Write down the general solution of the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, hence show that the expression for subsequent displacement for a string of length π fixed at both ends and released at rest with initial deflection $u(x, 0) = \sin x$ is $u(x, t) = \cos t \sin x$. (Given $c^2 = 1$) 1+3=4

UNIT—III

5. *(a)* Prove the Rodrigue's formula $P_n(x) = \frac{1}{2^n(n!)}\frac{d^n}{dx^n}(x^2 - 1)^n$ for Legendre's polynomials and hence show that $\int_{-1}^{1} P_0(x)\,dx = 2$; $(n = 0)$. 5+2=7

(b) Prove that $\frac{d}{dx}[J_0(x)] = -J_1(x)$; where $J_n(x)$ is the Bessel's function. 3

OR

6. *(a)* Starting from the expression $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$ for Bessel's function, prove the following recursion relations : 3+3=6

(i) $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

(ii) $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

(b) Starting from the Rodrigue's formula $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$ for Hermite polynomials, find the values of $H_3(x)$ and $H_4(x)$. 4

UNIT—IV

7. *(a)* Obtain the Fourier series for the function

$$f(x) = \begin{cases} \sin x \ ; & 0 \le x \le \pi \\ -\sin x \ ; & \pi \le x \le 2\pi \end{cases}$$

4

(b) Use the method of Fourier transform to solve $\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$; $x \ge 0$, $t \ge 0$ subject to the boundary conditions—*(i)* $u(0,t) = 0$, *(ii)* $u(x,0) = \begin{cases} 1 \ ; & 0 < x < 1 \\ 0 \ ; & x \ge 1 \end{cases}$ and *(iii)* $u(x,t)$ is bounded. 6

OR

8. *(a)* Obtain the Fourier integral of the function

$$f(x) = \begin{cases} 0 \ ; & x < 0 \\ \frac{1}{2} \ ; & x = 0 \\ e^{-x} \ ; & x > 0 \end{cases}$$

and hence show that $f(0) = \frac{1}{2}$. 3

(b) Find the infinite Fourier transform of the function $f(x) = \begin{cases} 1 \ , & |x| \le 1 \\ 0 \ , & |x| > 1 \end{cases}$ and hence show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. 4

(c) Find the inverse cosine transform of e^{-n}. 3

UNIT—V

9. *(a)* Use Laplace transform to solve the integral equation $F\ (t) \quad \sin t \quad {}_{0}^{t} F(t \quad x)\cos x\, dx$ for $F(t)$, if $F(0) \quad 0$. 4

(b) Using the method of partial fractions, find the inverse Laplace transform of $\dfrac{1}{(s \quad 1)(s^2 \quad 1)}$. 3

(c) Find the Laplace transform of $F(t) \quad t^2 e^{t} \sin 4t$. 3

OR

10. *(a)* Using Laplace transform, show that $x \quad e^{t} \cos t$ is the solution of the differential equation $\dfrac{d^2x}{dt^2} \quad 2\dfrac{dx}{dt} \quad 2x \quad 0$; given that $x_0 \quad (x\ )_{t \quad 0} \quad 1$. 4

(b) Using convolution theorem, find the inverse Laplace transform of

$$\frac{s^2}{(s^2 \quad a^2)^2}$$

3

(c) Find the Laplace transform of $F(t) \quad \cos^3 t$. 3

★ ★ ★