MATH/V/CC/08b **Student's Copy**

2022

(CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (B) PAPER

(Probability Theory)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick ☑ the correct answer in the boxes provided : 1×10=10

1. The chance that a leap year selected at random will contain 53 Sundays is

(a) $\frac{3}{7}$ ☐

(b) $\frac{2}{7}$ ☐

(c) $\frac{1}{7}$ ☐

(d) $\frac{5}{7}$ ☐

2. If $P(A)$ 0 25, $P(B)$ 0 15, $P(A$ $B)$ 0 30, then $P(A$ $B)$ is

(a) $\frac{2}{3}$ ☐

(b) 0·10 ☐

(c) 0·30 ☐

(d) None of the above ☐

3. For the binomial distribution

(a) mean < variance ☐

(b) variance < mean ☐

(c) mean = variance ☐

(d) None of the above ☐

4. For the probability mass function $f(x)$ $cx^2(1$ $x)$, 0 x 1, the value of the constant c is

(a) 5 ☐

(b) 8 ☐

(c) 12 ☐

(d) 0 ☐

5. The random variables X and Y with joint probability distribution $f(x, y)$ and marginal distribution $g(x)$ and $h(y)$ respectively are independent if and only if

(a) $f(x, y)$ $g(x)$ $h(y)$ ☐

(b) $f(x, y)$ $g(x) / h(y)$ ☐

(c) $f(x, y)$ $g(x)$ $h(y)$ ☐

(d) $f(x, y)$ $g(x) h(y)$ ☐

6. If $\text{var}(X) \quad 2$, then $\text{var}(3X \quad 4)$ is

(a) 18 ☐

(b) 22 ☐

(c) 32 ☐

(d) 10 ☐

7. For two random variables X and Y, $\text{var}(X \quad Y)$ is equal to

(a) $\text{var}(X) \quad \text{var}(Y)$ ☐

(b) $\text{var}(X) \quad \text{var}(Y)$ ☐

(c) $\text{var}(X) \quad \text{var}(Y) \quad 2\,\text{cov}(X, Y)$ ☐

(d) $\text{var}(X) \quad \text{var}(Y) \quad 2\,\text{cov}(X, Y)$ ☐

8. Two unbiased disc are thrown. If X is the sum of the numbers showing up, then $P[|X \quad 7| \quad 3]$ is

(a) $\frac{7}{3}$ ☐

(b) $\frac{5}{3}$ ☐

(c) $\frac{1}{3}$ ☐

(d) None of the above ☐

9. The mean and variance of the geometric distribution are

(a) $\frac{p}{q}, \frac{q}{p^2}$ ☐

(b) $\frac{p}{q}, \frac{p^2}{q}$ ☐

(c) $\frac{q}{p}, \frac{q}{p^2}$ ☐

(d) $\frac{q}{p}, \frac{p^2}{q}$ ☐

10. Let X and Y be independent random variables with $Z \quad X \quad Y$. Let $M_X(t)$, $M_Y(t)$ and $M_Z(t)$ be the moment generating functions of X, Y and Z respectively, then

(a) $M_Z(t) \quad M_X(t)\, M_Y(t)$ ☐

(b) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐

(c) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐

(d) $M_Z(t) \quad M_X(t) / M_Y(t)$ ☐

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. A bag contains 7 red, 12 white and 4 blue balls. What is the probability that three balls drawn at random are one of each colour?

OR

2. If A, B, C are mutually independent events, then show that $A \quad B$ and C are also independent.

UNIT—II

3. Determine the binomial distribution for which the mean is 4 and the variance is 3.

OR

4. If X is uniformly distributed over the interval $[a, b]$, then prove that $E(X) \quad \frac{a \quad b}{2}$.

UNIT—III

5. If X and Y are random variables having joint density function

$$f(x, y) \quad \begin{matrix} \frac{1}{8}(6 \quad x \quad y), & 0 \quad x \quad 2, 2 \quad y \quad 4 \\ 0 \quad , & \text{otherwise} \end{matrix}$$

find $P[X \quad 1 \quad Y \quad 3]$.

OR

6. The joint density function of X, Y is given as

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad y \quad 1 \\ 0, & \text{otherwise} \end{matrix}$$

Prove that X and Y are not independent random variables.

UNIT—IV

7. For random variables X and Y, prove that $E(X \quad Y) \quad E(X) \quad E(Y)$, provided all the expectation exist.

OR

8. If X is a random variable, then $V(aX \quad b) \quad a^2 V(X)$, where a and b are constants.

UNIT—V

9. Prove that the moment generating function of gamma distribution is

$$M_X(t) \quad (1 \quad t)^{\quad n}, \ |t| \quad 1$$

OR

10. If (X, Y) are independent Poisson variate such that $P(X \quad 1) \quad P(X \quad 2)$ and $P(Y \quad 2) \quad P(Y \quad 3)$, then find the variates of $(X \quad 2Y)$.

(Section : C—descriptive)

(*Marks* : 50)

Answer the following : 10×5=50

Unit—I

1. *(a)* A speaks the truth in 60% and B in 75% of the cases. In what percentage of the cases are they likely to contradict each other in starting the same fact? 5

(b) Prove that for any two events A and B,

$$P(A \quad B) \quad P(A) \quad P(A \quad B) \quad P(A) \quad P(B)$$ 5

OR

2. State and prove Bayes theorem. 10

Unit—II

3. For the binomial distribution $(q \quad p)^n$, prove that

$$_1 \quad pq \;\; nr \qquad \frac{d}{dp}$$

where is the rth central moment. Hence obtain $_2$, $_3$ and $_4$. 10

OR

4. *(a)* Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. 5

(b) A random variable X has the probability density function as follows :

$$f(x, y) \quad \begin{matrix} \frac{1}{4}, & 2 \quad x \quad 2 \\ 0, & \text{otherwise} \end{matrix}$$

Obtain the value of *(i)* $P(X \quad 1)$, *(ii)* $P(|X| \quad 1)$ and *(iii)* $P[(2X \quad 3) \quad 5]$. 5

UNIT—III

5. *(a)* For the bivariate probability distribution of X and Y

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	2/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

find the following : 6

(i) $P(X \quad 1, Y \quad 2)$

(ii) $P(X \quad 1)$

(iii) $P(Y \quad 3)$

(b) If

$$f(x, y) \quad \begin{matrix} \frac{2}{5}(2x \quad 3y), & 0 \quad x, y \quad 1 \\ 0 \quad , & \text{otherwise} \end{matrix}$$

then verify whether $f(x, y)$ is a joint probability distribution function. 4

OR

6. *(a)* The joint probability density function of two random variables X and Y is given by

$$f(x, y) \quad \frac{9(1 \quad x \quad y)}{2(1 \quad x)^4 (1 \quad y)^4}\ ; \ 0 \quad x, y$$

Find the marginal distribution of X and Y and the conditional distribution of Y for $X \quad x$. 6

(b) If X and Y are two random variables, determine whether X and Y are independent if

$$f(x, y) \quad \begin{matrix} 8xy, & 0 \quad y \quad x \quad 1 \\ 0 \ , & \text{otherwise} \end{matrix}$$

4

UNIT—IV

7. State and prove Chebyshev's inequality. 10

OR

8. Two random variables X and Y have the joint probability distribution function

$$f(x,\ y) \quad \begin{matrix} 2 \quad x \quad y, & 0 \quad x, y \quad 1 \\ 0 \quad , & \text{otherwise} \end{matrix}$$

Find the following : 10

(a) Marginal probability density function of X and Y

(b) $V(X)$ and $\text{cov}(X, Y)$

UNIT—V

9. *(a)* If X is a Poisson variate such that $P(X \quad 2) \quad 9P(X \quad 4) \quad 90P(X \quad 6)$, find the value of the parameter . 5

(b) Find the moment generating function of a normal distribution. 5

OR

10. *(a)* For a Poisson distribution, prove that

$$_{1} \quad r \quad _{1} \quad \frac{d}{d}$$

5

(b) Define geometric distribution for a random variable X. Find its mean and variance. 5

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