 Student's Copy

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(CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (A) PAPER

(Operations Research)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. In an LPP, the objective function is

(a) to be optimized ( )

(b) a linear function of decision variables ( )

(c) Both *(a)* and *(b)* ( )

(d) None of the above ( )

2. Graphical method can be applied to an LPP with

(a) two decision variables ( )

(b) three decision variables ( )

(c) more than two decision variables ( )

(d) None of the above ( )

3. In LPP, slack variable is added to the left hand side of the

(a) equation constraint ( )

(b) type constraint ( )

(c) type constraint ( )

(d) None of the above ( )

4. In simplex method, the optimality condition for an LPP in which objective function is to be maximized is that the net evaluation $Z_j \quad C_j$ is

(a) equal to zero for all j ( )

(b) 0 for all j ( )

(c) 0 for all j ( )

(d) Both *(a)* and *(b)* ( )

5. Dual simplex method can be applied to an LPP if

(a) initial basic solution is feasible ( )

(b) initial basic solution is not feasible ( )

(c) optimality condition is satisfied ( )

(d) initial basic solution is not feasible and optimality condition is satisfied ( )

6. In a transportation problem with m origins and n destinations, the number of basic variables is at most

(a) $m \quad n$ ( )

(b) $m \quad n \quad 1$ ( )

(c) mn ( )

(d) $mn \quad 1$ ( )

7. The LPP in which all the decision variables are to be integer valued is called

(a) pure integer linear programming problem ( )

(b) mixed integer linear programming problem ( )

(c) game theory problem ( )

(d) None of the above ( )

8. In cutting plane method, each cut involves the introduction of

(a) an equation constraint ( )

(b) a less than or equal to constraint with a negative number in the right hand side ( )

(c) an artificial variable ( )

(d) None of the above ( )

9. When there are two competitors playing a game, the game is called

(a) two-person game ( )

(b) fair game ( )

(c) strictly determinable game ( )

(d) None of the above ( )

10. If $\underline{V}$ and $\overline{V}$ be the maximin and minimax values of a game respectively, then

(a) $2\underline{V} \quad \overline{V}$ ( )

(b) $\underline{V} \quad 2\overline{V}$ ( )

(c) $\underline{V} \quad \overline{V}$ ( )

(d) $\underline{V} \quad \overline{V}$ ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Solve the following LPP by graphical method :

Maximize $Z \quad x_1 \quad x_2$

subject to the constraints

$$2x_1 \quad x_2 \quad 6$$
$$x_1 \quad 2x_2 \quad 6$$
$$\text{and } x_1, x_2 \quad 0$$

OR

2. An animal feed company must produce 200 kg of a mixture containing the ingredients X_1 and X_2. X_1 costs ₹ 3/kg and X_2 costs ₹ 8/kg. Not more than 80 kg of X_1 can be used and minimum quantity of X_2 to be used is 60 kg. Find how much of each ingredient should be used if the company wants to minimize the cost. Formulate the problem as an LPP.

UNIT—II

3. Solve the following system of linear equations using simplex method :

$$x \quad y \quad 7$$
$$x \quad y \quad 3$$

OR

4. Express the following LPP in standard form :

Maximize $Z \quad 2x_1 \quad x_2 \quad 4x_3$

Subject to the constraints

$$2x_1 \quad 4x_2 \quad 4$$
$$x_1 \quad 2x_2 \quad x_3 \quad 5$$
$$2x_1 \quad 3x_3 \quad 3$$

and $x_1, x_2 \quad 0$; x_3 unrestricted.

UNIT—III

5. Find the initial basic feasible solution of the following transportation problem :

Destination / *Origin*	*X*	*Y*	*Z*	*Supply*
A	5	1	8	12
B	2	4	6	14
C	3	6	7	4
Demand	9	10	11	

OR

6. Construct the dual of the following LPP :

Maximize $Z \quad 2x_1 \quad 3x_2$

subject to the constraints

$$x_1 \quad 2x_2 \quad 4$$
$$x_1 \quad x_2 \quad 6$$
$$x_1 \quad 3x_2 \quad 9$$
$$x_1,\ x_2 \quad 0$$

UNIT—IV

7. Solve the IPP :

Maximize $Z \quad 2x_1 \quad x_2$

subject to the constraints

$$6x_1 \quad 8x_2 \quad 24$$
$$x_1 \quad 2$$

$x_1,\ x_2 \quad 0$ and integers.

OR

8. The tableau of a maximization pure integer linear programming problem is given below :

	C_j		4	3	0	0
C_B	Basic variables	X_B	x_1	x_2	x_3	x_4
4	x_1	$\frac{6}{6}$	1	0	$\frac{1}{5}$	$\frac{2}{5}$
3	x_2	$\frac{21}{10}$	0	1	$\frac{2}{5}$	$\frac{3}{5}$
	Z_j		4	3	$\frac{2}{5}$	$\frac{7}{10}$
	$Z_j \quad C_j$		0	0	$\frac{2}{5}$	$\frac{7}{10}$

Find the Gomory's constraint.

UNIT—V

9. Solve the game with following payoff matrix :

Player B

Player A	5	1
	3	4

OR

10. Using dominance principles reduce the following payoff matrix to 2 2 matrix :

8	5	6
8	6	5
7	4	5
6	5	6

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. A diet is to contain at least 400 units of carbohydrates, 500 units of fat and 300 units of protein. Two foods *A* and *B* are available. *A* costs ₹ 2 per unit and *B* costs ₹ 4 per unit. A unit of food *A* contains 10 units of carbohydrate, 20 units of fat and 15 units of protein; a unit of food *B* contains 25 units of carbohydrates, 10 units of fat and 20 units of protein. Find the minimum cost for the diet that consists of mixture of these two foods and meets the minimum requirements. Formulate the problem as a linear programming problem and solve it by graphical method. 10

OR

2. *(a)* Use graphical method to solve the LPP given below : 7

$$\text{Minimize } Z \quad 4x_1 \quad 5x_2$$

subject to the constraints

$$2x_1 \quad 3x_2 \quad 12$$
$$x_1 \quad x_2 \quad 3$$
$$x_1 \quad 4$$
$$x_2 \quad 3$$

and $x_1, x_2 \quad 0$.

(b) Find the feasible region of the problem given below : 3

$$\text{Maximize } Z \quad 4x_1 \quad x_2$$

subject to the constraints

$$x_1 \quad x_2 \quad 2$$
$$5x_1 \quad 9x_2 \quad 45$$
$$x_2 \quad 4$$

and $x_1, x_2 \quad 0$.

UNIT—II

3. Solve the given LPP by using simplex method : 10

$$\text{Minimize } Z \quad 2x_1 \quad 3x_2 \quad 2x_3$$

subject to the constraints

$$2x_1 \quad 4x_2 \quad 12$$
$$3x_1 \quad x_2 \quad 2x_3 \quad 7$$
$$4x_1 \quad 3x_2 \quad 8x_3 \quad 10$$

and $x_1, x_2, x_3 \quad 0$.

OR

4. Solve the following LPP by Big-M method : 10

$$\text{Minimize } Z \quad x_1 \quad x_2 \quad x_3$$

subject to the constraints

$$x_1 \quad 4x_2 \quad 2x_3 \quad 5$$
$$3x_1 \quad x_2 \quad 2x_3 \quad 4$$

and $x_1, x_2, x_3 \quad 0$.

UNIT—III

5. Find the initial basic feasible solution using VAM. Also, obtain the optimum solution : 10

Origin \ *Destination*	D_1	D_2	D_3	D_4	*Supply*
O_1	3	7	6	4	5
O_2	2	4	3	2	2
O_3	4	3	8	5	3
Demand	3	3	2	2	2

OR

6. A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job is given in the following table :

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	18	14	28	32
	J_2	8	13	17	19
	J_3	10	15	19	22

What is the job assignment that will minimize the cost? 10

UNIT—IV

7. Use branch and bound method to solve the mixed IPP : 10

Maximize $Z \quad x_1 \quad x_2$

subject to the constraints

$$2x_1 \quad 5x_2 \quad 16$$
$$6x_1 \quad 5x_2 \quad 30$$

$x_1, x_2 \quad 0$ and x_2 integer.

OR

8. Use Gomory's cutting plane method to find the optimum integer solution to the IPP : 10

Maximize $Z \quad x_1 \quad 4x_2$

subject to the constraints

$$2x_1 \quad 4x_2 \quad 7$$
$$5x_1 \quad 3x_2 \quad 15$$

$x_1, x_2 \quad 0$ and integers.

UNIT—V

9. Using dominance principle, reduce the following payoff matrix :

Player B

		I	*II*	*III*	*IV*
Player A	*I*	18	4	6	4
	II	6	2	13	7
	III	11	5	17	3
	IV	7	6	12	2

Then solve the game using graphical method. 10

OR

10. Transform to LPP and solve the game problem whose payoff matrix is given below by using simplex method : 10

Player B

		B_1	B_2	B_3
Player A	A_1	1	1	1
	A_2	1	1	3
	A_3	1	2	1

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