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(CBCS)

(5th Semester)

MATHEMATICS

SEVENTH PAPER

(Complex Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the boxes provided : 1×10=10

1. If $\bar{z}$ is the conjugate of z, then

(a) $|z| \quad |\bar{z}|$ ☐

(b) $|z| \quad |\bar{z}|$ ☐

(c) $|z| \quad |\bar{z}|$ ☐

(d) $|z| \quad\quad |\bar{z}|$ ☐

2. If the principal argument, arg $z \quad\; /2$, then z lies on

(a) the positive imaginary axis ☐

(b) the positive real axis ☐

(c) the negative imaginary axis ☐

(d) the negative real axis ☐

3. For a function $f(z) \quad u \quad iv$ to be analytic, it must satisfy Cauchy-Riemann equation, which is

(a) $u_x \quad v_y, u_y \quad v_x$ ☐

(b) $u_x \qquad v_y, u_y \quad v_x$ ☐

(c) $u_x \quad v_y, u_y \qquad v_x$ ☐

(d) $u_x \qquad v_y, u_y \qquad v_x$ ☐

4. The derivative of $e^x(\cos y \quad i \sin y)$ with respect to x is

(a) e^x ☐

(b) e^y ☐

(c) $e^{x \quad iy}$ ☐

(d) $e^{x \quad iy}$ ☐

5. If $\lim\limits_{n} \left| \frac{u_{n \quad 1}}{u_n} \right| \quad l$, then the series $\quad u_n$ is convergent if

(a) $l \quad 0$ ☐

(b) $l \quad 0$ ☐

(c) $l \quad 1$ ☐

(d) $l \quad 1$ ☐

6. If $\quad a_n$ converges and $\quad |v_n \quad v_{n \quad 1}|$ converges, then

(a) $\quad (a_n \quad v_n)$ also converges ☐

(b) $\quad (a_n v_n)$ also converges ☐

(c) $\quad (a_n / v_n)$ also converges ☐

(d) $\quad (v_n / a_n)$ also converges ☐

7. If L is the curve L but traversed in opposite direction, then

(a) $_{L} f(z)\,dz \quad _{L} f(z)\,dz \quad 0$ ☐

(b) $_{L} f(z)\,dz \quad _{L} f(z)\,dz$ ☐

(c) $_{L} f(z)\,dz \quad _{L} f(z)\,dz$ ☐

(d) $_{L} f(z)\,dz \quad _{L} f(z)\,dz$ ☐

8. The value of $_{C} \dfrac{dz}{(z \quad z_0)}$, where C is the circle with center at z_0 and radius r is

(a) $i / 3$ ☐

(b) $i / 2$ ☐

(c) $2 \; i$ ☐

(d) $\log r$ ☐

9. A point at which the function $f(z)$ ceases to be analytic is called

(a) zero ☐

(b) singularity ☐

(c) limit point ☐

(d) residue ☐

10. If $f(z)$ is analytic and uniformly bounded in every domain, then

(a) $f(z)$ is always zero ☐

(b) $f(z)$ is discontinuous ☐

(c) $f(z)$ is constant ☐

(d) None of the above ☐

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Show that $\arg(z) \quad \arg(\overline{z}) \quad 2n$.

OR

2. Show that for two complex numbers z_1 and z_2, $|z_1 \quad z_2| \quad |z_1| \quad |z_2|$.

UNIT—II

3. Find the derivative of $r^n(\cos n \quad i \sin n \ )$.

OR

4. With suitable example, show that continuity is not a sufficient condition for existence of finite derivative.

UNIT—III

5. Show that every absolutely convergent series is a convergent series.

OR

6. Find the radius of convergence of $\quad 2^{\sqrt{n}} z^n$.

UNIT—IV

7. Give reasons why $\circ_C \dfrac{e^z}{z^2 \quad 9} dz \quad 0$, where C is a circle $C : |z| \quad 2$.

OR

8. Evaluate $\circ_C \dfrac{1}{z} dz$, where C is a circle $C : |z| \quad r$.

UNIT—V

9. What are the singular points of $f(z) \quad 1/(1 \quad e^{z})$? State whether they are isolated or non-isolated singularities.

OR

10. State Liouville's theorem.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* For any complex number z and w, prove that $|z|^2 w \quad |w|^2 z \quad z \quad w$ if and only if $z \quad w$ or $z\overline{w} \quad 1$. 5

(b) Prove that $|z_1 \quad z_2|^2 \quad |z_1 \quad z_2|^2 \quad 2|z_1|^2 \quad 2|z_2|^2$. 5

OR

2. *(a)* Discuss the representation of complex numbers by points of a sphere (stereographic projection). 5

(b) Show that the equation of a circle in the Argand plane can be put in the form $z\overline{z} \quad b\overline{z} \quad \overline{b}z \quad c \quad 0$, where c is real and b is a complex constant. 5

UNIT—II

3. *(a)* If $u \quad x^3 \quad 3xy^2 \quad 3x^2 \quad 3y^2 \quad 1$, then show that u is harmonic and find the analytic function $u \quad iv$. 5

(b) If n is real, then show that $r^n(\cos n \quad i \sin n \ )$ is analytic except when $r \quad 0$ and $n \quad 1$. 5

OR

4. *(a)* Show that the function $f(z) \quad \sqrt{|xy|}$ is not analytic at (0, 0) although Cauchy-Riemann equations are satisfied at that point. 5

(b) For an analytic complex function $f(z) \quad u(r, \quad) \quad iv(r, \quad)$, show that

$$\frac{u}{r} \quad \frac{1}{r}\frac{v}{} \quad \text{and} \quad \frac{u}{} \quad r\frac{v}{}$$ 5

UNIT—III

5. (a) Find the radii of convergence of the following series : 3+3=6

(i) $\frac{n!}{n^n} z^n$

(ii) $1 \quad \frac{1}{n} \quad ^{n^2} \quad z^n$

(b) Examine the behaviour of the power series $\frac{1}{n} z^n$ on the circle of convergence. 4

OR

6. (a) For what values of z does the series $\frac{1}{(z^2 \quad 1)^n}$ is convergent and find its sum. 5

(b) Show that the power series $a_n z^n$ and its derivative $n a_n z^{n \quad 1}$ have same radius of convergence. 5

UNIT—IV

7. (a) Evaluate by Cauchy integral formula

$$\oint_C \frac{dz}{(z^2 \quad 1)(z \quad 1)}$$

where C is the circle $|z| \quad 2$. 5

(b) Evaluate $\oint_C \frac{1}{z \quad z_0} dz$, where C is the curve $C : |z \quad z_0| \quad r$. 5

OR

8. *(a)* Evaluate $_{C}\frac{z \quad 2}{z}\,dz$, where C is the circle $|z| \quad 2$ from 0 to . 5

(b) Let $f(z)$ is analytic in a simply connected domain D and C is any closed curve lies inside D. Then prove that

$$_{C}\, f(z)\,dz \quad 0$$ 5

UNIT—V

9. State and prove Taylor's theorem for an analytic function. 10

OR

10. *(a)* Examine the nature of singularities of the following functions : 5

(i) $\frac{\cot \quad z}{(z \quad a)^2}$ at $z \quad a$ and z

(ii) $\operatorname{cosec}\frac{1}{z}$ at $z \quad 0$

(b) Obtain the Laurent's series expansion of the function

$$\frac{z \quad 3}{z(z^2 \quad z \quad 2)}$$

for the region $1 \quad |z| \quad 2$. 5

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