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(CBCS)

(5th Semester)

MATHEMATICS

FIFTH PAPER

(Computer Oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the boxes provided :

1. The value of , where , denote the forward and backward difference operator respectively is

(a) ☐ (b) ☐

(c) ☐ (d) ☐

2. If $f(x)$ ab^{cx}, then ${}^{2}f(x)$ is equal to

(a) $ab^{cx}(b^{ch}$ $1)$ ☐ (b) $(b^{ch}$ $1)^2 ab^{cx}$ ☐

(c) $(b^{ch}$ $1)^2 ab^{x}$ ☐ (d) $(b^{ch}$ $1)^2 abc^{cx}$ ☐

3. The relation between divided difference and simple difference is given by

(a) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{^n y_0}{n!h^n}$ ☐

(b) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{^n y_1}{n!h^n}$ ☐

(c) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{^n y_0}{(n \quad 1)!h^n}$ ☐

(d) $(x_n, \cdots, x_2, x_1, x_0) \quad \frac{^n y_1}{(n \quad 1)!h^n}$ ☐

where , denote divided and simple difference.

4. Newton's formula for forward interpolation formula is given by

(a) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\lfloor 2} \; ^2 y_0 \quad \cdots$ where $u \quad \frac{(x \quad x_n)}{h}$ ☐

(b) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\lfloor 2} \; ^2 y_0 \quad \cdots$ where $u \quad \frac{(x \quad x_0)}{h}$ ☐

(c) $y \quad y_n \quad u \; y_n \quad \frac{u(u \quad 1)}{\lfloor 2} \; ^2 y_n \quad \cdots$ where $h \quad \frac{(x \quad x_n)}{u}$ ☐

(d) $y \quad y_0 \quad u \; y_0 \quad \frac{u(u \quad 1)}{\lfloor 2} \; ^2 y_0 \quad \cdots$ where $h \quad \frac{(x \quad x_0)}{u}$ ☐

5. Indirect method of solving a simultaneous linear equation be represented by

(a) Gauss elimination method ☐
(b) Gauss-Jordan method ☐
(c) Gauss-Seidel method ☐
(d) None of the above ☐

6. Solving the system of simultaneous equations in n unknowns by Crout's method, the method involves :

(i) diagonal matrix
(ii) upper and lower triangular matrices
(iii) forward and backward substitutions
(iv) diagonally dominant matrix

Which of the following is true?

(a) (i), (ii) and (iii) ☐
(b) (i) and (ii) only ☐
(c) (ii) and (iii) only ☐
(d) (i) and (iv) only ☐

7. When numerical integration is applied for the integration of a function of single variable, the method is called

(a) trapezoidal rule ☐ *(b)* general quadrature ☐

(c) Simpson's 1/3rd rule ☐ *(d)* mechanical quadrature ☐

8. From general quadrature formula, we can obtain a variate formula by putting n 1, 2, 3, $\cdots$. The best is found for

(a) n 2 only ☐ *(b)* n 2 and 6 both ☐

(c) n 2 and 4 both ☐ *(d)* n 1 only ☐

9. Which of the following statements is wrong?

(a) For solving ordinary differential equation numerically, Euler's method needs h to be very large to get a reasonable accuracy. ☐

(b) Euler's method is the Runge-Kutta method of first-order. ☐

(c) For solving ordinary differential equation numerically, the most reliable and most accurate method is Runge-Kutta method. ☐

(d) Modified Euler's method is Runge-Kutta method of second-order. ☐

10. For solving ordinary differential equation numerically, which among the following is applied if successive integration can be obtained easily?

(a) Euler's method ☐ *(b)* Taylor's method ☐

(c) Picard's method ☐ *(d)* Runge-Kutta method ☐

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

UNIT—I

1. Derive the relation $^2[f(x \quad 2h)] \quad ^2f(x)$, where and are the backward and forward difference operator respectively.

OR

2. Write Newton-Raphson formula to find the cube root of N. Hence find the cube root of 10.

UNIT—II

3. Find the divided difference of $[u, v, w]$ for the function $f(a) \quad \frac{1}{a^2}$.

OR

4. Show that the divided differences are independent of the order of arguments i.e., $(x_0,\ x_1) \quad (x_1,\ x_0)$. Is it true for more than two arguments also?

UNIT—III

5. Solve the given equation by Gauss-Jordan method :

$$x \quad 2y \quad 4,\ x \quad y \quad 13$$

OR

6. Reduce the system $3x \quad 9y \quad 2z \quad 10;\ 4x \quad 2y \quad 13z \quad 19;\ 4x \quad 2y \quad z \quad 3$ into diagonally dominant and write the corresponding system of equation.

UNIT—IV

7. Find the value of $\log 2^{\frac{1}{3}}$ from $\ _0^1 \frac{x^2}{1 \quad x^3}\,dx$ using Simpson's $\frac{1}{3}$rd rule with $h \quad 0\ 25$.

OR

8. Obtain the formula for the first-order derivatives $\frac{dy}{dx}$ or $f\ (x)$ for numerical differentiation.

UNIT—V

9. Using Euler's method, solve $\frac{dy}{dx} \quad x \quad y$ with $y(0) \quad 1$ and find $y(0\ 4)$ by taking $h \quad 0\ 2$.

OR

10. Find the value of $y(0\ 1)$ by Picard's method, given $\frac{dy}{dx} \quad \frac{y \quad x}{y \quad x}$, $y(0) \quad 1$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Find the function whose first difference is $x^3 \quad 3x^2 \quad 5x \quad 12$. 5

(b) Use the method of successive iteration to find the root of $3x \quad \log_{10} x \quad 6$. 5

OR

2. *(a)* Write the algorithm of Regula-Falsi method for finding the root of the equation $f(x) \quad 0$. 5

(b) If $y \quad \frac{1}{x(x \quad 3)(x \quad 6)}$, then prove that $^2 y \quad \frac{108}{x(x \quad 3)(x \quad 6)(x \quad 9)(x \quad 12)}$. 5

UNIT—II

3. *(a)* Find the cubic polynomial from the following data using Newton's divided difference formula : 4

x	0	1	2	5
$f(x)$	2	3	12	147

(b) Obtain Newton's backward interpolation formula for interpolation with equal intervals of the argument. 6

OR

4. *(a)* Interpolate the following function as a quadratic polynomial using Lagrange's interpolation formula, and find $f(10)$: 4

x	1	7	15
y	168	192	336

(b) Obtain Newton's divided difference interpolation formula for non-equal intervals of the argument. 6

UNIT—III

5. *(a)* Solve the following system of equations by Gaussian elimination method : 4

$$3x \quad y \quad 2z \quad 3,\ 2x \quad 3y \quad z \quad 3,\ x \quad 2y \quad z \quad 4$$

(b) Solve the following system by Gauss-Seidel method : 6

$$2x \quad y \quad 3;\ 2x \quad 3y \quad 5$$

OR

6. *(a)* Solve the following system of equation by Crout's method $x \quad y \quad z \quad 1$, $3x \quad y \quad 3z \quad 5$ and $x \quad 2y \quad 5z \quad 10$. 6

(b) Write an algorithm of Gauss elimination method for solving system of simultaneous linear equation. 4

UNIT—IV

7. *(a)* Evaluate $\,^{1}_{0} \frac{dx}{1 \quad x^2}$ using trapezoidal rule with $h \quad 0 \; 2$. Hence determine the value of . 4

(b) Find the second derivative of $f(x)$ at $x \quad 3 \; 0$ from the following table : 6

x	3·0	3·2	3·4	3·6	3·8	4·0
y	14·000	10·032	5·296	0·256	6·672	14·000

OR

8. *(a)* Find the first derivative of $f(x)$ at $x \quad 0 \; 4$ from the following table : 4

x	0·1	0·2	0·3	0·4
y	1·10517	1·22140	1·34986	1·49182

(b) Obtain the formula for Simpson's one-third rule for numerical integration. 6

UNIT—V

9. *(a)* Using Taylor's method, find $y(0 \; 1)$ correct to 3 decimal places from

$$\frac{dy}{dx} \quad 2xy \quad 1, \; y_0 \quad 0$$ 4

(b) Compute $y(0 \; 1)$ by Runge-Kutta method of fourth-order for the differential equation

$$\frac{dy}{dx} \quad xy \quad y^2, \quad y(0) \quad 1$$ 6

OR

10. Solve $\frac{dy}{dx} \quad y$ with $y(0) \quad 1$ by using Milne's method $x \quad 0 \; 1$ to $x \quad 2 \; 7$ with $h \quad 0 \; 3$. 10

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